Greatest Hits: Algebra

Warm-Up

Math Circle Competition Team

February 19th, 2017

1. **(2004 AMC 12B)** A function f is defined by $f(z) = i\overline{z}$, where $i = \sqrt{-1}$ and $\overline{z}$ is the complex conjugate of z. How many values of z satisfy both $|z| = 5$ and $f(z) = z$?

2. **(2008 AMC 12A)** Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

3. **(2017 AMC 12A)** There are 24 different complex numbers z such that $z^{24} = 1$. For how many of these is z^6 a real number?

4. **(2011 AMC 12B)** The arithmetic mean of two distinct positive integers x and y is a two-digit integer. The geometric mean of x and y is obtained by reversing the digits of the arithmetic mean. What is $|x - y|$?

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In-Class Problems

Math Circle Competition Team

February 19th, 2017

1. **(2005 AIME II)** For how many positive integers n less than or equal to 1000 is $(\sin t + i\cos t)^n = \sin nt + i\cos nt$ true for all real t?

2. **(2014 AIME I)** Let $x_1 < x_2 < x_3$ be the three real roots of the equation $\sqrt{2014}x^3 - 4029x^2 + 2 = 0$. Find $x_2(x_1 + x_3)$.

3. **(2013 AIME I)** There are nonzero integers a, b, r, and s such that the complex number $r + si$ is a zero of the polynomial $P(x) = x^3 - ax^2 + bx - 65$. For each possible combination of a and b, let $p_{a,b}$ be the sum of the zeros of $P(x)$. Find the sum of the $p_{a,b}$'s for all possible combinations of a and b.

4. **(2004 AIME II)** Let S be the set of integers between 1 and 2^{40} whose binary expansions have exactly two 1's. If a number is chosen at random from S, the probability that it is divisible by 9 is p/q, where p and q are relatively prime positive integers. Find $p + q$.

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Take-Home Problem Set

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Week of February 19th, 2017

1. **(2010 AMC 12A)** Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

2. **(2010 AMC 12A)** The number obtained from the last two nonzero digits of 90! is equal to n. What is n?