AMC Practice Problem Set

Math Circle Competition Team

January 21st, 2018

Algebra

1. **(2009 AMC 10A #9)** Positive integers a, b, and 2009, with $a < b < 2009$, form a geometric sequence with an integer ratio. What is a?

2. **(2009 AMC 10B #21)** What is the remainder when $3^0 + 3^1 + 3^2 + \cdots + 3^{2009}$ is divided by 8?

3. **(2007 AMC 10A #17)** Suppose that m and n are positive integers such that $75m = n^3$. What is a minimum possible value of $m + n$?

4. **(2009 AMC 12A #9)** Suppose that $f(x + 3) = 3x^2 + 7x + 4$ and $f(x) = ax^2 + bx + c$. What is $a + b + c$?

5. **(2008 AMC 12B #12)** For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008th term of the sequence?

6. **(2007 AMC 12A #18)** The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has real coefficients, and $f(2i) = f(2 + i) = 0$. What is $a + b + c + d$?

Combinatorics / Probability

1. **(2008 AMC 10B #16)** Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, the sum is 0.)

2. **(2007 AMC 10B #20)** A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

3. **(2009 AMC 10A #25)** For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

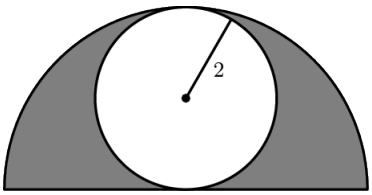
4. **(2009 AMC 12A #12)** How many positive integers less than 1000 are 6 times the sum of their digits?

5. **(2007 AMC 12B #13)** A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?

6. **(2009 AMC 12B #21)** Ten women sit in 10 seats in a line. All of the 10 get up and then reseat themselves using all 10 seats, each sitting in the seat she was in before or a seat next to the one she occupied before. In how many ways can the women be reseated?

Geometry

1. **(2009 AMC 10A #6)** A circle of radius 2 is inscribed in a semicircle, as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?



2. **(2007 AMC 12B #14)** Point P is inside equilateral $\triangle ABC$. Points Q, R, and S are the feet of the perpendiculars from P to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Given that $PQ = 1$, $PR = 2$, and $PS = 3$, what is AB?

3. **(2008 AMC 10A #16)** Points A and B lie on a circle centered at O, and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

4. **(2009 AMC 12A #14)** A triangle has vertices $(0, 0)$, $(1, 1)$, and $(6m, 0)$, and the line $y = mx$ divides the triangle into two triangles of equal area. What is the sum of all possible values of m?

5. **(2008 AMC 12A #18)** Triangle ABC, with sides of length 5, 6, and 7, has one vertex on the positive x-axis, one on the positive y-axis, and one on the positive z-axis. Let O be the origin. What is the volume of tetrahedron $OABC$?

6. **(2007 AMC 12B #19)** Rhombus $ABCD$, with side length 6, is rolled to form a cylinder of volume 6 by taping $\overline{AB}$ to $\overline{DC}$. What is $\sin(\angle ABC)$?

Potpourri

1. **(2009 AMC 10B #6)** Kiana has two older twin brothers. The product of their three ages is 128. What is the sum of their three ages?

2. **(2008 AMC 10A #11)** While Steve and LeRoy are fishing 1 mile from shore, their boat springs a leak, and water comes in at a constant rate of 10 gallons per minute. The boat will sink if it takes in more than 30 gallons of water. Steve starts rowing toward the shore at a constant rate of 4 miles per hour while LeRoy bails water out of the boat. What is the slowest rate, in gallons per minute, at which LeRoy can bail if they are to reach the shore without sinking?

3. **(2007 AMC 10B #16)** A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

4. **(2009 AMC 12A #15)** For what value of n is $i+2i^2+3i^3+\cdots+ni^n = 48+49i$?

5. **(2008 AMC 12B #10)** Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?

6. **(2007 AMC 12A #13)** A piece of cheese is located at $(12, 10)$ in a coordinate plane. A mouse is at $(4, -2)$ and is running up the line $y = -5x + 18$. At the point (a, b) the mouse starts getting farther from the cheese rather than closer to it. What is $a + b$?