

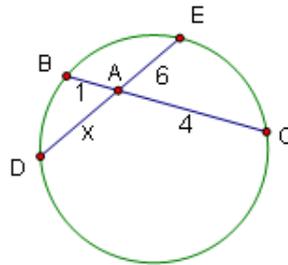
Power of a Point Theorem

Warm-Up

Math Circle Competition Team

September 11th, 2016

1. Find the value of x in the following diagram:



2. Chords AB and CD of a given circle are perpendicular to each other and intersect at a right angle at point E . Given that $BE = 16$, $DE = 4$, and $AD = 5$, find CE .
3. Square $ABCD$ of side length 10 has a circle inscribed in it. Let M be the midpoint of \overline{AB} . Find the length of that portion of the segment \overline{MC} that lies outside of the circle.
4. (2012 HMMT Nov.) ABC is a triangle with $AB = 15$, $BC = 14$, and $CA = 13$. The altitude from A to BC is extended to meet the circumcircle of ABC at D . Find AD .

Power of a Point Theorem

In-Class Problems

Math Circle Competition Team

September 11th, 2016

Law of Cosines:

1. Two boats are initially at the same location in the ocean. The first travels due south at 4 km/h; the second proceeds 60° east of north at 3 km/h. If the first boat has enough fuel for 5 hours of traveling, what is the maximum distance south the first boat's captain can travel, so that the fuel remaining will allow a safe return to the second boat? (You may assume Earth is flat.)

2. (2003 AIME) In convex quadrilateral $ABCD$, $\angle A \cong \angle C$, $AB = CD = 180$, and $AD \neq BC$. The perimeter of $ABCD$ is 640. Find $\cos A$.

Power of a Point:

1. (2013 AMC 12A) In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects \overline{BC} at points B and X . Moreover \overline{BX} and \overline{CX} have integer lengths. What is BC ?

2. **(2014 HMMT Feb.)** Let O_1 and O_2 be concentric circles with radii 4 and 6, respectively. A chord AB is drawn in O_1 with length 2. Extend AB to intersect O_2 in points C and D . Find CD .
3. **(2013 HMMT Nov.)** Points A, B, C lie on a circle ω such that BC is a diameter. AB is extended past B to point B' and AC is extended past C to point C' such that line $B'C'$ is parallel to BC and tangent to ω at point D . If $B'D = 4$ and $C'D = 6$, compute BC .
4. **(2013 HMMT Feb.) Challenge.** In triangle ABC , $\angle A = 45^\circ$ and M is the midpoint of BC . AM intersects the circumcircle of ABC for the second time at D , and $AM = 2MD$. Find $\cos \angle AOD$, where O is the circumcenter of ABC .

Power of a Point Theorem

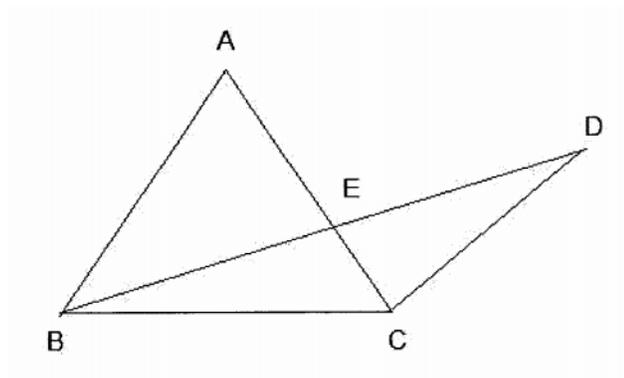
Take-Home Problem Set

Math Circle Competition Team

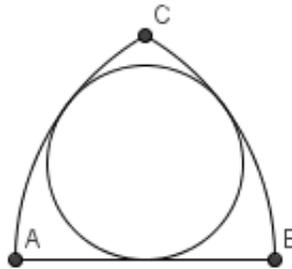
Week of September 11th, 2016

If you get stuck, we can answer your questions and give you hints on the forum at forum.mathcircle.us. Solutions will be posted on Saturday, September 17th.

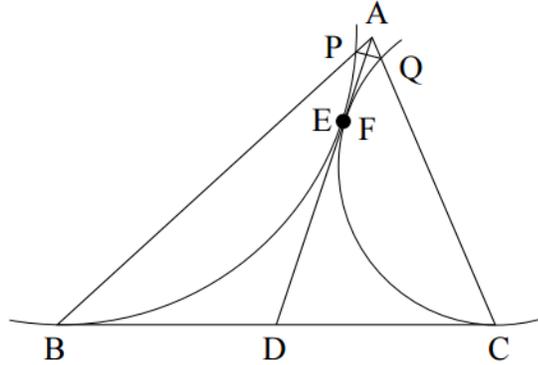
1. (2006 PUMaC) Given that in the diagram below, $\angle ACB = 65^\circ$, $\angle BAC = 50^\circ$, $\angle BDC = 25^\circ$, $AB = 5$, and $AE = 1$, determine the value of $BE \cdot DE$.



2. (2000 AMC 12) If circular arcs AC and BC have centers at B and A , respectively, then there exists a circle tangent to both \widehat{AC} and \widehat{BC} , and to \overline{AB} . If the length of \widehat{BC} is 12, find the circumference of the circle.



3. (2003 HMMT Feb.) In triangle ABC , $\angle ABC = 50^\circ$ and $\angle ACB = 70^\circ$. Let D be the midpoint of side BC . A circle is tangent to BC at B and is also tangent to segment AD ; this circle intersects AB again at P . Another circle is tangent to BC at C and is also tangent to segment AD ; this circle intersects AC again at Q . Find $\angle APQ$ (in degrees).



4. (2005 AIME I) Challenge. Triangle ABC has $BC = 20$. The incircle of the triangle (the circle inscribed in the triangle) evenly trisects the median AD . If the area of the triangle is $m\sqrt{n}$ where m and n are integers and n is not divisible by the square of a prime, find $m + n$.

