Important Inequalities

Warm-Up

Math Circle Competition Team

November 20th, 2016

1. If $xyz = 27$ and x, y, and z are positive real numbers, find the minimum value of $x + y + z$.

2. **(2006 AMC 12B)** In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 15. What is the greatest possible perimeter of the triangle?

3. **(2006 AIME II)** The lengths of the sides of a triangle with positive area are $\log_{10} 12$, $\log_{10} 75$, and $\log_{10} n$, where n is a positive integer. Find the number of possible values for n.

4. Over all triplets of positive real numbers x, y, and z satisfying $x + y + z = 3$, find the minimum value of

$$f(x, y, z) = \frac{yz + 4zx + 9xy}{xyz}.$$

5. **(1958 AHSME)** Find the maximum value of $\dfrac{x^2 - 2x + 2}{2x - 2}$ for $-4 < x < 1$.

6. **(2013 AMC 12B)** Let a, b, and c be real numbers such that

$$a + b + c = 2, \text{ and}$$

$$a^2 + b^2 + c^2 = 12$$

What is the difference between the maximum and minimum possible values of c?

Important Inequalities

In-Class Problems

Math Circle Competition Team

November 20th, 2016

1. **(1983 AIME)** Find the minimum value of $\dfrac{9x^2 \sin^2 x + 4}{x \sin x}$ for $0 < x < \pi$.

2. If $x + 2y = 1$ and x and y are positive real numbers, find the minimum possible value of $\dfrac{1}{x} + \dfrac{1}{y}$.

3. **(2014 HMMT Nov. Team)** The side lengths of a triangle are distinct positive integers. One of the side lengths is a multiple of 42, and another is a multiple of 72. What is the minimum possible length of the third side?

4. If x, y, and z are positive real numbers satisfying $xyz = 32$, find the minimum value of

$$x^2 + 4xy + 4y^2 + 2z^2.$$

5. A wooden box with volume 250 cubic feet is to be built, and for support the bottom side must be three times thicker than the other sides. Find the dimensions that minimize the amount of wood used. How many square feet of wood are needed? (Do not use calculus!)

6. **(2014 AMC 12B)** Real numbers a and b are chosen with $1 < a < b$ such that no triangle with positive area has side lengths 1, a, and b or $\frac{1}{b}$, $\frac{1}{a}$, and 1. What is the smallest possible value of b?

Important Inequalities

Take-Home Problem Set

Math Circle Competition Team

Week of November 20th, 2016

If you get stuck, we can answer your questions and give you hints on the forum at ***forum.mathcircle.us***. Solutions will be posted on Saturday, November 26th.

1. **(2005 HMMT Feb. Algebra)** If $a, b, c > 0$, what is the smallest possible value of $\left\lfloor \frac{a+b}{c} \right\rfloor + \left\lfloor \frac{b+c}{a} \right\rfloor + \left\lfloor \frac{c+a}{b} \right\rfloor$? (Note that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

2. **(1998 Putnam) Challenge.** Find the minimum value of

$$\frac{(x+1/x)^6 - (x^6+1/x^6) - 2}{(x+1/x)^3 + (x^3+1/x^3)}.$$